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Maximum Likelihood Signal-to-Noise Ratio Estimation for AWGN Channels with 1-bit ADCs

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***Abstract*—Future communications systems with very high bandwidths face challenges with the power consumption of the analog-to-digital converters (ADCs). Reducing the amplitude resolution to 1 bit is a promising solution, which, however, introduces new challenges in digital baseband processing, with one of them being the estimation of the signal-to-noise ratio (SNR). To this end, we study maximum likelihood SNR estimation for systems employing 1-bit ADCs. Hereby, we assume the transmission of known pilot sequences over an additive white Gaussian noise channel. We evaluate the Cramér-Rao lower bound and derive a novel closed-form maximum likelihood estimator, including a sequential estimation approach. Furthermore, we investigate the benefits of employing uniform constant modulus pilots, which are equivalent to uniform phase dithering, and present a practical estimator for this case. Finally, we evaluate the performance of the proposed estimators numerically.**

***Index Terms*—1-bit ADCs, Cramér-Rao lower bound, maximum likelihood, signal-to-noise ratio estimation**

I. Introduction

To meet the demand for ever-increasing data rates for 6G and beyond, future communications systems are anticipated to utilize the vast amount of available spectrum in the sub-THz bands above 100 GHz [1]. In practice, this results in a significantly increased analog-to-digital converter (ADC) power consumption, which has been shown in [2], stating that the ADC power consumption grows quadratically with the Nyquist sampling rate for sampling rates above 1 GHz, different to lower sampling rates, where the increase is only linear. Therefore, it is expected that the ADC power consumption becomes a bottleneck in upcoming sub-THz wideband systems. A promising approach to mitigate the ADC bottleneck is to reduce the amplitude resolution down to 1-bit per real dimension. The inherent loss in achievable data rate due to 1-bit quantization can partly be compensated by temporal oversampling [3]. Practical transceiver designs for systems employing 1-bit temporal oversampling ADCs have been proposed in [4], [5]. Such systems, similar to conventional systems, require signal-to-noise ratio (SNR) knowledge to implement detection [4]–[6]. Hence, it is of great interest how to estimate the SNR without amplitude information being available.

Maximum likelihood (ML) SNR estimation for additive white Gaussian noise (AWGN) channels with infinite amplitude resolution has been studied in [7]–[9]. The works [8], [9] also study the corresponding Cramér-Rao lower bound (CRLB). A comprehensive overview of SNR estimation for AWGN channels and a performance comparison of different estimators can be found in [10].

Regarding channel parameter estimation with 1-bit quantization previous works include an upper bound on the CRLB with 1-bit ADCs [11], which is extended to complex-valued signals in [12]. Also, bounds on phase, frequency, timing, and phase noise estimation were considered in [13] and [14]. Similarly, upper bounds on the CRLB for joint SNR and time-of-arrival estimation using 1-bit temporal oversampling ADCs have been derived and evaluated in [15]. Moreover, ML parameter estimation from 1-bit quantized observations of linear models has been studied in [16], and in [17] a weighted partial decision detector is employed to estimate the SNR by correlating the 1-bit quantized receive signal with a local reference signal.

In this work, we study SNR estimation for AWGN channels with a receiver employing 1-bit quantization. To this end, we assume the transmission of a known pilot sequence, i.e., we focus on data-aided estimators. In the unquantized case, this problem involves estimating two parameters, i.e., the signal and noise power, respectively, cf. [10], whereas in the 1-bit quantized case, the problem only involves a single parameter, i.e., the SNR. Moreover, we derive an analytic expression of the exact CRLB in this case and, different to [17], we derive a novel closed-form expression of the ML estimator when employing quadrature phase shift keying (QPSK) pilot symbols. To enable reliable SNR estimation at high SNR, we improve the ML estimator further by introducing a sequential ML estimation approach by allowing a variable observation length. As an alternative, we also derive a novel approximate ML estimator and its corresponding CRLB for pilot symbols that are uniformly distributed on the unit circle, to which we refer as uniform constant modulus (UCM) pilots. This is equivalent to uniformly dithering the phase at the input of the 1-bit ADC. Subsequently, we compare the performance of the proposed SNR estimation approaches by numerical evaluation.

Notation: In this work, random quantities are written in

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sans-serif letters, e.g., x, whereas x denotes their corresponding realization. We use j, $\mathrm{Re}(\cdot)$, $\mathrm{Im}(\cdot)$, $\mathbb{E}[\cdot]$, and $\mathrm{Q}(\cdot)$ to denote the imaginary unit, the real part, the imaginary part, stochastic expectation, and the Gaussian Q-function, respectively.

II. System Model

We consider the linearly modulated transmit signal

$$\mathsf{s}(t) = \sum_{l=1}^{K} \mathsf{x}_l h(t - lT) \tag{1}$$

where the transmit symbols $\mathbf{x} = [\mathsf{x}_1, ..., \mathsf{x}_K]^\mathsf{T}$ are a sequence of independent and identically distributed (i.i.d.) random variables. Throughout this work, we consider two different uniform distributions of the transmit symbols. In the first case, the symbols x_k are chosen from a symbol alphabet corresponding to a unit energy QPSK modulation with equal symbol probabilities, and in the second case, the transmit symbols are distributed uniformly on the unit circle, i.e., $\arg(\mathsf{x}_k) \sim \mathcal{U}(0, 2\pi)$, which we abbreviate as UCM transmit symbols. Moreover, $h(t)$ denotes the impulse response of a root-raised-cosine (RRC) transmit filter with the single-sided bandwidth $W_h = \frac{\alpha+1}{2T}$, with the symbol duration T, and the roll-off factor $\alpha \in [0, 1]$.

At the receiver, the signal is filtered with an RRC filter with bandwidth $W_r = W_h$ and sampled at symbol rate, i.e., $\frac{1}{T_\mathrm{s}} = \frac{1}{T}$, resulting in the unquantized receive samples

$$\mathsf{r}_k = \sqrt{S}\,\mathsf{s}_k + \sqrt{N}\mathsf{n}_k, \quad k = 1, \ldots, K \tag{2}$$

with $\mathsf{s}_k = \mathsf{s}(kT_\mathrm{s})$. The channel introduces zero-mean circularly-symmetric Gaussian noise with independent real and imaginary parts. After receive filtering and sampling the noise is given by

$$\mathbf{n} = [\mathsf{n}_1, ..., \mathsf{n}_K]^\mathsf{T} \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{I}_K) \tag{3}$$

with identity matrix $\boldsymbol{I}_K$ of size $K \times K$. Furthermore, we define the SNR as

$$\rho = \frac{\lim_{T'\to\infty} \frac{1}{T'} \int_{-T'/2}^{T'/2} |\sqrt{S}s(t)|^2 \mathrm{d}t}{\mathbb{E}_{\mathsf{n}_k}\left[|\sqrt{N}\mathsf{n}_k|^2\right]} = \frac{S}{N} \tag{4}$$

where $S > 0$ and $N > 0$ denote the signal power and the noise variance, respectively. In this work, determining the SNR ρ is the main objective, for which, however, only samples from a 1-bit ADC are available. The 1-bit quantized receive samples are denoted by

$$\mathsf{y}_k = \mathrm{csign}(\mathsf{r}_k) = \mathrm{sign}(\mathrm{Re}(\mathsf{r}_k)) + j\,\mathrm{sign}(\mathrm{Im}(\mathsf{r}_k)) \tag{5}$$

where we define the signum function as $\mathrm{sign}(x) = 1$ for $x \geq 0$ and $\mathrm{sign}(x) = -1$ otherwise. For notational convenience, we define the vectors $\boldsymbol{x} = [x_1, \ldots, x_K]^\mathsf{T}$, $\boldsymbol{s} = [s_1, \ldots, s_K]^\mathsf{T}$, $\boldsymbol{y} = [y_1, \ldots, y_K]^\mathsf{T}$, and $\boldsymbol{r} = [r_1, \ldots, r_K]^\mathsf{T}$.

III. Cramér-Rao Lower Bound

Before investigating the CRLB, we study the likelihood function of the SNR given the 1-bit quantized observations.

Proposition 1. *The likelihood function $p(\boldsymbol{y}|\boldsymbol{x}; S, N)$ can be expressed as a function of the ratio $\rho = \frac{S}{N}$, i.e., it holds that*

$$p(\boldsymbol{y}|\boldsymbol{x}; S, N) = p(\boldsymbol{y}|\boldsymbol{x}; \rho). \tag{6}$$

This unique property follows directly from the likelihood function of the 1-bit quantized observations. Note that a similar result does not hold for the unquantized observations $\boldsymbol{r}$, i.e., $p(\boldsymbol{r}|\boldsymbol{x}; S, N) \neq p(\boldsymbol{r}|\boldsymbol{x}; \rho)$. Therefore, in the 1-bit quantized case, the estimation of the SNR ρ reduces to a single-parameter estimation problem, whereas in the unquantized case, it is necessary to estimate two parameters (cf. [10] and references therein).

Proposition 2. *The CRLB for estimating the SNR ρ from the 1-bit quantized observations $\boldsymbol{y}$ of fixed length K is given by*

$$\mathrm{CRLB}_K(\rho) = 4\pi\rho \left[\sum_{k=1}^{K} \left(\frac{\exp(-2\rho\,\mathrm{Re}^2(x_k))\mathrm{Re}^2(x_k)}{Q(\sqrt{2\rho}\,\mathrm{Re}(x_k))Q(-\sqrt{2\rho}\,\mathrm{Re}(x_k))} + \frac{\exp(-2\rho\,\mathrm{Im}^2(x_k))\mathrm{Im}^2(x_k)}{Q(\sqrt{2\rho}\,\mathrm{Im}(x_k))Q(-\sqrt{2\rho}\,\mathrm{Im}(x_k))} \right) \right]^{-1}. \tag{7}$$

Equation (7) is obtained directly from the definition of the Fisher information (FI) $F(\rho) = -\mathbb{E}_\mathbf{y}\left[\frac{\partial^2}{\partial\rho^2} \log(p(\mathbf{y}|\boldsymbol{x}; \rho))\right]$ and then derived from [12, eq. (26)] using the independence of the observations and $\mathrm{CRLB}(\rho) = (F(\rho))^{-1}$ [18, Th. 3.1].

After obtaining an expression for the CRLB for arbitrary transmit symbols, we specify the result to QPSK pilots and to UCM pilots in the following two corollaries.

Corollary 1. *When employing QPSK pilot symbols, i.e., in the case $x_k \in \frac{\pm 1 \pm j}{\sqrt{2}}$, and the number of observations is fixed to K, the CRLB given in* (7) *simplifies to*

$$\mathrm{CRLB}_{K,\mathrm{QPSK}}(\rho) = \frac{4\pi\rho}{K} \exp(\rho)\, Q(\sqrt{\rho})Q(-\sqrt{\rho}). \tag{8}$$

Corollary 2. *In case of employing UCM pilots, the CRLB* (7) *can be very tightly approximated by*

$$\mathrm{CRLB}_{K,\mathrm{UCM}}(\rho) \approx \frac{4\pi\rho}{K} \frac{e^{c_2\rho}}{c_1\big(I_0(c_2\rho) - I_1(c_2\rho)\big)} \tag{9}$$

with $I_\nu(\cdot)$ being the modified Bessel function of the first kind of order ν, $c_1 = 4.0360$, and $c_2 = 0.3930$.

The approximation (9) follows directly from [13, eq. (62)] due to $\arg(\mathsf{x}_k) \sim \mathcal{U}(0, 2\pi)$.

Finally, in the following proposition, we evaluate the loss due to 1-bit quantization at low-SNR, which is well-known, see e.g., [12], [15].

Proposition 3. *The low-SNR loss due to 1-bit quantization is given by*

$$\lim_{\rho\to 0} \frac{\mathrm{CRLB}_K(\rho)}{\mathrm{CRLB}_\mathrm{uq}(\rho)} = \frac{\pi}{2} \tag{10}$$

where $\mathrm{CRLB}_\mathrm{uq}(\rho) = \frac{\rho}{K}(\rho + 2)$ (cf. [10, eq. (64)]) denotes the CRLB for the unquantized system.

Prop. 3 is obtained by calculating the limit

$$\lim_{\rho\to 0} \frac{u\exp(-2\rho u)}{Q(\sqrt{2\rho u})Q(-\sqrt{2\rho u})} = 4u \tag{11}$$

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and then utilizing (11) to compute the limits of the real parts and the imaginary parts in (7), respectively.

IV. Maximum Likelihood SNR Estimation

In this section, we consider ML estimation of the SNR based on the 1-bit quantized observations $\boldsymbol{y}$, which can be expressed as

$$\hat{\rho}_{\mathrm{ML}} = \underset{\rho>0}{\operatorname{argmax}}\; p(\boldsymbol{y}|\boldsymbol{x};\rho). \tag{12}$$

A. QPSK Pilot Symbols

1) Fixed Observation Length Estimation: Here, we focus on ML estimation when employing QPSK pilot symbols with a fixed observation length K. In this case, we obtain a closed-form ML estimator, as stated in the following theorem.

Theorem 1. *When employing QPSK pilot symbols, the ML estimator of the SNR ρ, based on the 1-bit quantized observations $\boldsymbol{y}$ of length K, is given by*

$$\hat{\rho}_{\mathrm{ML}} = \left(Q^{-1}\left(\frac{1}{8K}||\boldsymbol{y}-\sqrt{2}\boldsymbol{x}||_2^2\right)\right)^2 \tag{13}$$

where $Q(\cdot)$ denotes the tail distribution function of the standard normal distribution.

Proof. First, we note that when employing QPSK pilots and sampling at symbol rate, there is an analytic description of a one-to-one mapping between the bit error rate (BER) P_{b} and the SNR ρ, i.e., $P_{\mathrm{b}} = Q\left(\sqrt{\rho}\right)$ [19, Sec. 5-2-7]. The inverse mapping is given by

$$\rho = \left(Q^{-1}\left(P_{\mathrm{b}}\right)\right)^2. \tag{14}$$

Then, it can be shown that the observed BER $\hat{P}_{\mathrm{b}}$ can be expressed as

$$\hat{P}_{\mathrm{b}} = \frac{1}{8K}||\boldsymbol{y}-\sqrt{2}\boldsymbol{x}||_2^2 \tag{15}$$

and corresponds to the ML estimate of the true BER P_{b}. Finally, (13) follows from the invariance property [18, Th. 7.2] of the ML estimator, i.e., by inserting the ML estimate $\hat{P}_{\mathrm{b}}$, given in (15), into (14). □

After obtaining the ML estimator, we investigate its bias and mean square error (MSE) in the following lemma and corollary.

Lemma 2. *For finite K and for $\rho < \infty$, the ML SNR estimator $\hat{\rho}_{\mathrm{ML}}$, given in* (13)*, has infinite bias, i.e., $\mathbb{E}_{\mathbf{y}|\rho}[\hat{\rho}_{\mathrm{ML}} - \rho] = \infty$.*

Proof. For finite K, the ML BER estimator $\hat{P}_{\mathrm{b}}$, given in (15), can only take discrete values from the set $\mathcal{A} = \left\{\frac{0}{2K}, \frac{1}{2K}, \ldots, \frac{2K}{2K}\right\}$ with $|\mathcal{A}| = 2K+1$. Then, the mean of the ML SNR estimator is lower-bounded by

$$\begin{aligned}\mathbb{E}_{\mathbf{y}|\rho}\left[\hat{\rho}_{\mathrm{ML}}\right] &= \sum_{a\in\mathcal{A}} \mathrm{P}(\hat{P}_{\mathrm{b}}=a)\left(Q^{-1}(a)\right)^2 \\ &\geq \mathrm{P}(\hat{P}_{\mathrm{b}}=0)\left(Q^{-1}(0)\right)^2 \overset{(a)}{=} \infty\end{aligned} \tag{16}$$

where (a) is due to the fact that $\mathrm{P}(\hat{P}_{\mathrm{b}}=0) = (1-P_{\mathrm{b}})^{2K} > 0$. Utilizing (16) as well as the condition $\rho < \infty$ concludes the proof. □

Corollary 3. *For finite K and for $\rho < \infty$, the MSE of the ML SNR estimator, given in Thm. 1, is infinite, i.e.,*

$$\mathbb{E}_{\mathbf{y}|\rho}\left[(\hat{\rho}_{\mathrm{ML}} - \rho)^2\right] = \infty. \tag{17}$$

The corollary follows directly from Lemma 2 and from

$$\begin{aligned}\mathbb{E}_{\mathbf{y}|\rho}[(\hat{\rho}_{\mathrm{ML}}-\rho)^2] = \mathbb{E}_{\mathbf{y}|\rho}[(\hat{\rho}_{\mathrm{ML}}-\mathbb{E}_{\mathbf{y}|\rho}[\hat{\rho}_{\mathrm{ML}}])^2] \\ + \mathbb{E}^2_{\mathbf{y}|\rho}[\hat{\rho}_{\mathrm{ML}}-\rho].\end{aligned} \tag{18}$$

Note that the SNR estimator in [17] also employs a metric based on the BER and likewise the estimator fails at high SNR.

2) Sequential SNR Estimation: Due to its infinite bias and infinite MSE, we modify the ML estimator by allowing the number of observations to increase sequentially until a reliable estimate is obtained. In this setting, the number of observed bit errors will be fixed to the constant C. Similar to (14), the estimate for the SNR ρ will then be given by

$$\hat{\rho}_{\mathrm{seq}} = \left(Q^{-1}\left(\hat{P}_{\mathrm{b}}\right)\right)^2 = \left(Q^{-1}\left(\frac{C}{2L}\right)\right)^2 \tag{19}$$

where, however, the number of samples L is a realization of the random variable L. Due to sampling at symbol rate, the observations are i.i.d., and the probability of observing a bit error is given by $P_{\mathrm{b}} = Q(\sqrt{\rho})$, whereas observing no bit error has the probability $1 - P_{\mathrm{b}} = Q(-\sqrt{\rho})$. Therefore, the distribution of L can be expressed as

$$P(L;\rho,C) = \binom{2L-1}{C-1}\left(Q(\sqrt{\rho})\right)^C\left(Q(-\sqrt{\rho})\right)^{2L-C} \tag{20}$$

which is the probability mass function of the negative binomial distribution. The estimator (19) is also the ML estimator for ρ, which can be proven in a similar way as Thm. 1.

Next, we calculate the CRLB parameterized by the number of observed bit errors C for the sequential estimator, by utilizing the distribution of the number of observation samples (20), its expectation $\mathbb{E}_{\mathsf{L}}[\mathsf{L}] = C/(2Q(\sqrt{\rho}))$, and its variance $\mathbb{E}_{\mathsf{L}}\left[(\mathsf{L}-\mathbb{E}[\mathsf{L}])^2\right] = CQ(-\sqrt{\rho})/(4Q^2(\sqrt{\rho}))$ [20, Sec. 3.2]. From those, we calculate the FI $I(\rho) = \mathbb{E}_{\mathsf{L}}\left[\left(\partial/\partial\rho \log P(\mathsf{L};\rho,C)\right)^2\right]$ to obtain the CRLB for a variable observation length as

$$\mathrm{CRLB}_C(\rho) = \frac{8\pi\rho}{C}\exp(\rho)\,Q^2(\sqrt{\rho})\,Q(-\sqrt{\rho}). \tag{21}$$

Note that this result is equal to the CRLB for a fixed observation length (8) with K replaced by $\mathbb{E}_{\mathsf{L}}[\mathsf{L}] = C/(2Q(\sqrt{\rho}))$.

This sequential estimation approach avoids the problem of undefined estimates at high SNR. For low SNR, however, the sequential estimator is likely to terminate very early, in the extreme case after $2L = C$ observations, i.e., $\hat{P}_{\mathrm{b}} = 1$, for which the SNR estimate in (19) is undefined.

3) Hybrid SNR Estimation: By combining both the fixed observation length and the sequential estimation approach, we introduce a hybrid SNR estimator, which has the requirement of a minimum number of observations K as well as a minimum number of observed bit errors C. In this section, we denote the vector of the 1-bit quantized observations of length $L \geq K$ by $\boldsymbol{y}^L$ to specify its adaptive length. Furthermore, let

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$\hat{\rho}_{\mathrm{ML}} = f(\boldsymbol{y}^L)$ be the ML estimator given by (13). The hybrid SNR estimator is then defined as

$$\hat{\rho}_{\mathrm{hyb}} = g(\boldsymbol{y}^L) = \begin{cases} f(\boldsymbol{y}^L), & \text{if } \frac{1}{4}||\boldsymbol{y}^L - \sqrt{2}\boldsymbol{x}^L||_2^2 \geq C \\ g(\boldsymbol{y}^{L+1}), & \text{otherwise} \end{cases} \tag{22}$$

which we initialize with $\boldsymbol{y}^K$.

The hybrid SNR estimator circumvents the disadvantages of both the fixed-length estimator at high SNR and the sequential estimator at low SNR.

B. UCM Pilot Symbols

When employing UCM pilot symbols, the ML estimator cannot be expressed in closed form, as the mapping between the SNR ρ and the BER P_{b} depends on the phase $\varphi_k = \arg(x_k)$ of each single pilot symbol x_k and is given by

$$P_{\mathrm{b}} = \frac{1}{2K}\sum_{k=1}^{K}\Big(Q\big(\sqrt{2\rho}\cos(\varphi_k)\big) + Q\big(\sqrt{2\rho}\sin(\varphi_k)\big)\Big). \tag{23}$$

Therefore, in this section, an approximation for the ML estimator for UCM pilot symbols is derived. To this end, we approximate (23) by averaging P_{b} over φ_k. Using that $\varphi_k \sim \mathcal{U}(0, 2\pi)$, (23) converges for a large number of observations K towards its expectation, i.e.,

$$P_{\mathrm{b}} \xrightarrow{p} \mathbb{E}_{\varphi_k}\left[\frac{1}{2}Q\big(\sqrt{2\rho}\cos(\varphi_k)\big) + \frac{1}{2}Q\big(\sqrt{2\rho}\sin(\varphi_k)\big)\right] \tag{24}$$

$$= \bar{P}_b = \frac{1}{2\pi}\int_0^1 \mathrm{erfc}(\sqrt{\rho\, u})(1-u)^{-\frac{1}{2}}u^{-\frac{1}{2}}\,\mathrm{d}u \tag{25}$$

where (25) follows from the substitutions $u = \cos^2(\varphi_k)$ in the first and $u = \sin^2(\varphi_k)$ in the second addend inside the expected value operator, respectively. Expressing the complementary error function in terms of the confluent hypergeometric function, i.e., $\mathrm{erfc}(z) = 1 - \frac{2z}{\sqrt{\pi}}{}_1F_1(\frac{1}{2}; \frac{3}{2}; -z^2)$ enables the application of the general Euler transform [21, eq. (4.1.2)], yielding the analytic expression for the expected bit error rate

$$\bar{P}_b = \frac{1}{2} - 2\sqrt{\frac{\rho}{\pi^3}}{}_2F_2\left(\frac{1}{2}, 1; \frac{3}{2}, \frac{3}{2}; -\rho\right) \tag{26}$$

which has the form $\bar{P}_b = g(\rho)$. However, to obtain the approximate ML SNR estimate

$$\hat{\rho}_{\mathrm{AML}} = g^{-1}(\hat{P}_{\mathrm{b}}) \tag{27}$$

the inverse function $\rho = g^{-1}(\bar{P}_{\mathrm{b}})$ is required. As $\hat{P}_{\mathrm{b}}$ has the finite support $\{\frac{0}{2K}, \frac{1}{2K}, ..., \frac{2K}{2K}\}$, g^{-1} can be efficiently implemented via a lookup table, where the value $\hat{P}_{\mathrm{b}} = 0$ is mapped to a large but finite SNR. Note that (26) is independent of the specific realization of the pilot sequence. Therefore, the lookup table only needs to be calculated once, whereas (23) needs to be recalculated for each realization of the pilot sequence.

V. Numerical Results

In this section, we evaluate the performance of the proposed SNR estimators and compare them to their corresponding CRLBs. First, we analyze the CRLBs for QPSK and UCM pilot symbols for a fixed number of observations K, displayed in Fig. 1 as the normalized CRLB (NCRLB),

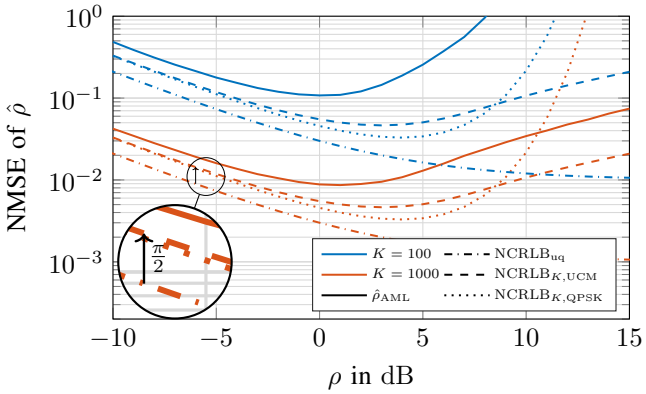

Fig. 1. NCRLBs for QPSK and UCM pilot symbols and fixed observation length K, normalized to true SNR, and estimation performance for UCM pilot symbols over ρ

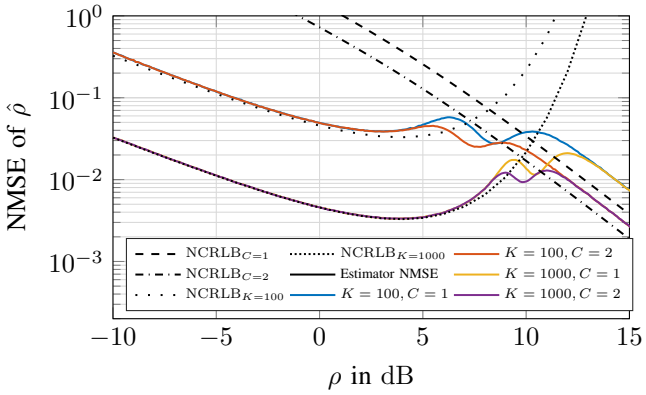

Fig. 2. Estimation performance of the hybrid SNR estimator, NCRLB_K for fixed observation length, NCRLB_C for fixed number of errors, for QPSK pilots over ρ

i.e., $\mathrm{NCRLB}(\rho) = \mathrm{CRLB}(\rho)/\rho$. At low SNR, the normalized CRLB decreases with ρ, with a loss of $\pi/2$ relative to the unquantized case, verifying Prop. 3. At medium SNR, between 0 dB and 8 dB, the CRLB for UCM pilot symbols is slightly higher than for QPSK pilot symbols, while at higher SNR, both bounds rise significantly. The bound for UCM pilot symbols, however, increases substantially slower, since the pilot symbols can be closer to the decision threshold of the 1-bit ADC, leading to an increased number of bit errors at high SNR, which improves the estimation performance. Moreover, the uniform distribution of the UCM pilots is equivalent to dithering the phase at the input of the quantizer, reducing the effect of the non-linearity of the 1-bit quantization. A practical realization of the phase dither can be implemented by sampling at a low intermediate frequency [13].

Next, we evaluate the performance of the proposed SNR estimators. As the MSE of the ML estimator for QPSK with a fixed number of observations is infinite, it is omitted here. The normalized MSE (NMSE) of the approximate ML UCM estimator is shown in Fig. 1. It does not attain the CRLB for UCM pilot symbols due to the employed approximations, but achieves a better performance than QPSK-based estimation at high SNR for a fixed sample size K.

In Fig. 2, the estimation performance of the hybrid SNR estimator for QPSK pilot symbols is displayed together with the normalized CRLB_K for a fixed number of pilot symbols K in

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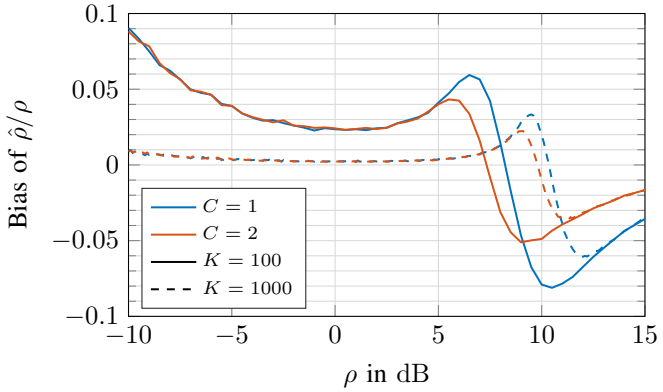

Fig. 3. Normalized bias of the hybrid SNR estimator over ρ with minimum number of observations K and minimum number of observed errors C

(8) and the normalized CRLB_C for a fixed number of observed errors C in (21). The hybrid SNR estimator combines both of those cases. At low SNR ($\leq 5\,\mathrm{dB}$), it is very likely that the estimator terminates at fixed K with a variable number of errors, and its MSE approaches the CRLB_K. Whereas, at high SNR, beyond $12\,\mathrm{dB}$, likely the minimum number of bit errors C is not reached within K observations, so the sequential approach is used, terminating at fixed C with a variable number of observations L. Here, the MSE is then close to the CRLB_C. In the region of a medium SNR between $5\,\mathrm{dB}$ and $12\,\mathrm{dB}$ the normalized MSE first increases again and deviates from the normalized CRLB_K before crossing it. Then, the estimator is able to reach a lower MSE than both CRLBs simultaneously, as both of the two conditions on the minimum number of pilot symbols K and the minimum number of observed errors C have to be fulfilled, while the bounds are derived for each condition separately. Note that achieving the displayed low normalized MSEs at high SNR requires the transmission of very long pilot sequences, i.e., the variable number of observations L becomes large.

Moreover, Fig. 3 shows the normalized bias of the hybrid SNR estimator for QPSK pilot symbols. At low SNR, where the hybrid estimator operates in the fixed observation length mode, the bias vanishes with increasing minimum observation length K. However, in the region of $\mathrm{SNR} > 5\,\mathrm{dB}$, the normalized bias increases. Even though here, the estimator utilizes a variable number of observations L which is larger than K there is a remaining bias. Note that this is likely the reason for the gap of the MSE to the CRLB_C at high SNR in Fig. 2.

VI. Conclusion

In this work, we considered the SNR estimation based on 1-bit quantized samples for AWGN channels. Contrary to the case with infinite amplitude resolution, the estimation of the SNR with 1-bit quantized samples is a single-parameter estimation problem. We derived the CRLB for the general case of a data-aided SNR estimator and provided closed-form expressions for QPSK pilots and UCM pilots, where UCM pilots outperform QPSK pilots at high SNR. We derive the ML estimator and a sequential ML estimator for QPSK pilots, both of which we combine to benefit from both their advantages. For the case of UCM pilots, we propose a practical approximation for the ML SNR estimator, which, compared to QPSK pilots, shows superior achievable performance at high SNR but performs worse at low SNR. Overall, the presented estimation algorithms are able to determine the SNR from 1-bit quantized observations, enabling the subsequent implementation of a detection stage in receivers with 1-bit ADCs.

References

[1] T. S. Rappaport, Y. Xing *et al.*, “Wireless communications and applications above 100 GHz: Opportunities and challenges for 6G and beyond,” *IEEE Access*, vol. 7, pp. 78 729–78 757, June 2019.

[2] B. Murmann, “ADC Performance Survey 1997-2025,” [Online]. Available: https://github.com/bmurmann/ADC-survey.

[3] L. Landau, M. Dörpinghaus, and G. Fettweis, “1-bit quantization and oversampling at the receiver: Sequence-based communication,” *EURASIP J. Wireless Commun. Netw.*, vol. 2018, no. 1, Apr. 2018.

[4] R. Deng, J. Zhou, and W. Zhang, “Bandlimited communication with one-bit quantization and oversampling: Transceiver design and performance evaluation,” *IEEE Trans. Commun.*, vol. 69, no. 2, pp. 845–862, Feb. 2021.

[5] P. Neuhaus, M. Dörpinghaus, and G. Fettweis, “Zero-crossing modulation for wideband systems employing 1-bit quantization and temporal oversampling: Transceiver design and performance evaluation,” *IEEE Open J. Commun. Soc.*, vol. 2, pp. 1915–1934, 2021.

[6] S. Zeitz, P. Neuhaus *et al.*, “Soft-output equalizers for systems employing 1-bit quantization and temporal oversampling,” in *Proc. IEEE Wireless Commun. Netw. Conf. (WCNC)*, Nanjing, China, Mar. 2021.

[7] R. B. Kerr, “On signal and noise level estimation in a coherent PCM channel,” *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-2, no. 4, pp. 450–454, July 1966.

[8] T. R. Benedict and T. T. Soong, “The joint estimation of signal and noise from the sum envelope,” *IEEE Trans. Inf. Theory*, vol. 13, no. 3, pp. 447–454, July 1967.

[9] C. M. Thomas, “Maximum likelihood estimation of signal-to-noise ratio,” Ph.D. dissertation, USC, Los Angeles, CA, USA, June 1967.

[10] D. Pauluzzi and N. Beaulieu, “A comparison of SNR estimation techniques for the AWGN channel,” *IEEE Trans. Commun.*, vol. 48, no. 10, pp. 1681–1691, 2000.

[11] M. Stein, A. Mezghani, and J. A. Nossek, “A lower bound for the Fisher information measure,” *IEEE Signal Process. Lett.*, vol. 21, no. 7, pp. 796–799, 2014.

[12] M. Schlüter, M. Dörpinghaus, and G. Fettweis, “Bounds on channel parameter estimation with 1-bit quantization and oversampling,” in *Proc. IEEE Int. Workshop Signal Process. Advances Wireless Commun. (SPAWC)*, Kalamata, Greece, June 2018.

[13] ——, “Bounds on phase, frequency, and timing synchronization in fully digital receivers with 1-bit quantization and oversampling,” *IEEE Trans. Commun.*, vol. 68, no. 10, Oct. 2020.

[14] S. Zeitz, F. Gast *et al.*, “On the Bayesian Cramér-Rao bound for phase noise estimation based on 1-bit quantized samples,” in *Proc. IEEE Global Commun. Conf. (GLOBECOM 2020)*, Rio de Janeiro, Brazil, Apr. 2022.

[15] M. S. Stein, “Performance analysis for time-of-arrival estimation with oversampled low-complexity 1-bit A/D conversion,” in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP)*, New Orleans, LA, USA, Mar. 2017, pp. 4491–4495.

[16] J. Zhu, X. Wang *et al.*, “Maximum likelihood estimation from sign measurements with sensing matrix perturbation,” *IEEE Trans. Signal Process.*, vol. 62, no. 15, pp. 3741–3753, Aug. 2014.

[17] L. Yuanling and Q. Xilong, “SNR estimation arithmetic base on 1 bit quantization correlator,” *IEEE Access*, vol. 12, pp. 175 839–175 842, Nov. 2024.

[18] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Upper Saddle River, NJ, USA: Prentice-Hall, Inc., 1993.

[19] J. G. Proakis, *Digital Communications*. New York, NY, USA: McGraw-Hill, 2008.

[20] G. Casella and R. L. Berger, *Statistical Inference*. Pacific Grove, CA, USA: Duxbury, 2002.

[21] L. J. Slater, *Generalized Hypergeometric Functions*. London, England: Cambridge University Press, 1966.

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